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VELOCITY FIELD AT THE WALL OF VESSEL WITH AXIAL MIXER AND RADIAL BAFFLES

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In this work is studied the velocity field along the wall of a cylindrical vessel with radial baffles at mixing by an axial mixer in turbulent regime. After finding the distribution of velocity along the vessel wall the flow pattern is constructed for flow between two neighbouring baffles. This flow pattern is found to be independent of the relative size of the mixer and vessel. From the obtained results is considered the heat transfer intensity from the wall into the charge for various configurations of the mixed system.

Mixing of liquids by mechanical mixers is advantageous especially in cases when the intensification of heat transfer between the reactor wall and reacting mixture is necessary. For this purpose are in cases of high speed rotational mixers very often used axial mixers with blades forcing the liquid in direction of their axis either upward or downward. Liquid is in this way brought to motion which usually has in the whole system a turbulent character. Aim of this work is to study experimentally the velocity field at the wall of a vessel with radial baffles. From these results can be constructed the flow pattern between two neighbouring baffles and made quantitative conclusions concerning the effect of velocity field on intensity of heat transfer from the wall into the mixed charge.

The study of velocity field at the wall of mixed vessel with baffles was made by Askew and Beckmann[1]. Their system was a cylindrical vessel of 254 mm diameter provided with baffles and with mixers of three types: Turbine, four-bladed paddle with inclined blades, and three-bladed propeller mixer ($s = d$) with diameters 76 and 102 mm. Height of the mixer above the vessel bottom equalled to one third of the liquid level in the vessel when at rest, which was equal to the vessel diameter. Velocity profiles in radial rays at the vessel wall were determined from values of pressure measurements made by Pitot tube of 1·25 mm diameter, with a hole diameter 0·50 mm in the cylindrical wall of the tube[2]. In a close vicinity of the vessel wall (*i.e.* at the distan-

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ce up to 4 mm from the wall) was found an insignificant velocity maximum and the flow direction was found practically identical with the direction of vertical axis of the mixed system. The maximum velocity in the measured radial profile was found directly proportional to the rotational speed of the mixer.

A measurement of the velocity field in the mixed charge can be made in many ways[3]. Principally at the vessel wall come into consideration three basic methods[4]: The hot-wire anemometer, flow visualization, and pressure tube. The first of the mentioned methods cannot be used in the considered region of the mixed system as until now not fully comprehended methods of measurement in liquids were developed, while the use of air as a model fluid was, for the studied case, from the technical point of view impossible. Visualization of the flow can be used for determining the flow direction in a given point at the wall of a solid body[5]. By visual tracing of a hair or of a thread tied to a solid body can be — at the place where the thread is tied — determined the direction of the flow. From knowledge of the direction flow at the wall in a given point and from the determined distribution of total and static pressure by use of the Pitot tube and of a static tube[3,6] can be determined the velocity profile close to the wall of a solid body. At these conditions, however, is very high the value of the component of velocity gradient which is perpendicular to the body surface which is consequently causing a certain error in determining the total pressure by Pitot tube. To reduce this error it is advisable to use an empirical relation proposed by Young and Mass[7] for correction of the place of measurements of total pressure by Pitot tube in the direction of decreasing velocity gradient, *i.e.* in the direction from the wall of the solid body. Generally, it is necessary to minimalize the dimensions of a probe used for measuring the velocity profile at the wall of a solid body so that its effect on the studied velocity field be as little as possible.

EXPERIMENTAL

Apparatus

The apparatus consists of two parts: The drive with the mixer and the vessel with the charge and measuring equipment. *The drive* consists of a direct current electric motor with a power input of 0·5 kW with a magnetic control of rotational speed. Electric motor with the gears was fixed in a special stand to reduce the effect of vibration on the measuring system. Three-bladed propeller mixer was used ($s = d$)[8] of diameter d equal to 1/3, 1/4, and 1/5 of the vessel diameter D, and a six-bladed paddle mixer[8] with a slope of blades 45°, with diameter d equal to 1/3 of the vessel diameter. The mixer was always placed centrally in the vessel in 1/3 of the height between the bottom and the liquid level when at rest, and it rotated so that the blades forced the liquid toward the vessel bottom.

The cylindrical vessel with flat bottom of diameter $D = 290$ mm was made of perspex and was provided with four radial baffles of the width equal to 1/10 of the vessel diameter[3,8]. On the vessel wall were marked points between two neighbouring baffles, in which vicinity were made the pressure measurements (see Fig. 1). On the opposite side of the vessel, at points equivalent to points of pressure measurements were sticked into the vessel wall black linen threads of length 15 mm without affecting the wall surface. The threads served for determination of the liquid flow direction in the given point of the wall. As a charge was used distilled water at a temperature 20°C. Its level height when at rest H equalled to the vessel diameter. The used construction enabled motion of the probe in the mixed system both in radial and axial directions. Its movements were made by screw with lead 1 mm. The probe attachement was of a brass tube, with a revolving holder on its lower end enabling partial rotation of the probe round its axis and its movement in the axial-radial plane to the mixer axis. Through the extension tube of the probe and through the holder were lead plastic tubes of inside diameter 1 mm connecting the probe with the manometer. The extension tube was furthermore revolving round its own axis. The probe was connected with the plastic tubes by stainless steel tubes. The velocity field at the wall was measured by three probe types: *1*. The radial profile of the over-all pressure in adjacent vicinity of the wall was measured by a two-hole Pitot tube (see Fig. 2*a*) with the inside diameter of each hole $d_1 = 0.4$ mm and with the outside diameter $d_2 = 0.8$ mm. By this tube, the value of over-all pressure at two points of the measured profile could have been taken at the same time. *2*. For determination of radial profile of the static pressure in adjacent vicinity of the wall was used a static tube (see Fig. 2*b*). *3*. For determination of velocity profile in the region between two neighbouring baffles at constant radial distance 4 mm from the wall, was used the three-hole oriented Pitot tube which had been described in the previous paper of this series[3]. This type of probe was used as at these points the radial velocity component could not be neglected, as in the adjacent vicinity of the wall.

Measurements

The temperature of the charge was kept at 20°C with an accuracy $\pm 1°$. For a six-bladed paddle mixer with inclined blades, the velocity field was measured at two levels of rotational speeds and for each size of the propeller mixer at three values of rotational speeds of the mixer, which was kept by magnetic controller on the set value with an accuracy $\pm 1\%$. This quantity was measured by the photoelectric measuring instrument with the accuracy of one revolution. Height of the liquid level in the vessel when at rest and the distance of the mixer above centre of the bottom was measured by a steel ruler with the accuracy ± 0.5 mm. Time intervals in measuring the rota-

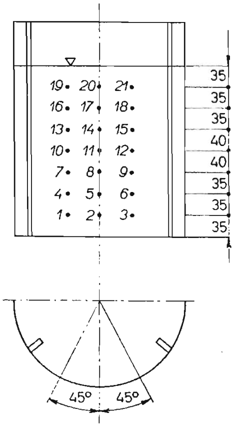

FIG. 1

Distribution and Notation of Positions of Measurement of Local Velocity Vector at the Vessel Wall

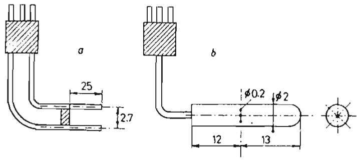

FIG. 2

Pitot Tube

a) Two-hole, *b*) static tube.

tional speeds of the mixer had the accuracy of ± 0.1 s. Direction of velocity vector were determined from the angle of thread deviation from the vertical direction in the plane tangential to the cylindrical area in the given point at the wall. This angle was measured by protractor on a stand so that in each point of measurement (see Fig. 1) was the angle of inclination β of the thread taken 16-times (in half-minute intervals) with an accuracy of $\pm 5°$, with the coordinate system chosen according to Fig. 3. Beside angle β was, in measuring the flow direction in a given point, determined the estimate $\tilde{\varphi}$ of the angle φ, *i.e.* the angle of thread deviation in the axial-radial plane (see Fig. 3). This angle had to be known for positioning of the three-hole Pitot tube.

Main part of the measurements made was the determination of pressures by instruments described above. Two series of measurements were done: Determination of velocity profile in radial direction in adjacent vicinity of the wall, and determination of velocity in various points at constant distance from the wall between two neighbouring baffles.

Local value of static pressure was measured by static tube facing the flow direction in a considered point located by use of the indication thread. Holes in the tube walls were then on the radial ray between the considered point on the wall and the system axis. The tube was connected with the inclined manometer filled with distilled water. As reference was considered the hydrostatic pressure in the point of the tube, when the mixer was at rest. Local value of static pressure was measured in two positions on the radial ray at the distance of the tube axis 2·0 and 5·0 mm from the wall. The tube could be set to a required position with an accuracy ± 0.05 mm (as the initial position was considered the contact point of the tube with the vessel determined visually by mirroring of the tube on the alighted vessel wall). *Radial profile of the overall pressure* at the vessel wall was measured by a two-hole Pitot tube (see Fig. 2*a*) so that this tube was oriented in direction opposite to the flow found in the given point at the wall by the indication thread (angle $\bar{\beta}$). The measuring holes of the tube were then on the radial ray between the considered position on the wall and the system axis. Each of the tubes was connected with the inclined manometer filled with distilled water. As referential was considered the hydrostatic pressure when the mixer was at still. Values of pressure on the manometers connected with individual tube holes were taken after five minutes. Their level was considered steady when in two successive readings they did not differ by more than ± 0.5 mm which is also the accuracy of readings on the used manometers. By the used tube it was possible to measure the local value of the over-all pressure in two points on a given radial ray simultaneously. The distance of these points of measurement equals to the distance of axes of tube taps (see Fig. 2*a*). This distance ensures that one tube at measurement of the over-all pressure will not be affected by the other one and *vice versa*[9]. Local value of over-all pressure was thus measured in all in this positions on the radial ray at the wall at the distance 0·40 to 5·10 mm from the wall. These series were performed in positions *4–6*, *10–12*, and *16–18* at the wall between two neighbourint baffles (see Fig. 1). It was possible to set the mentioned probe into the required distance from the wall with an accuracy ± 0.05 mm in the same way as the static tube. The mentioned accuracy of measurements of individual quantities enables determination of value of the velocity vector with a relative error about 20% for velocities lower than 0·3 m/s and about 10% for higher velocities. In general it can be said that the relative error of determination decreases with the increasing velocity.

Local value of over-all pressure was measured in region where it can be assumed that the flow is not affected by the wall effect, by a three-hole oriented Pitot tube[3] having the central hole always 4 mm perpendicularly from the wall (on the radial ray) and side holes 3 resp. 5 mm from the wall. The tube was always situated in the given point (see Fig. 1) in accordance with the formerly determined angles $\tilde{\varphi}$ and $\bar{\beta}$, *i.e.* opposite to the flow at a given point.* Differences of over-all

* Construction of all Pitot tubes[3] used ensured measurement of over-all pressure in a given point independently of the angle β within the range of $\pm 25°$.

pressures Δp_j [$j = 1, 2, 3$] were measured with manometers connected to individual tube taps with reference values with the mixer at rest. Accuracy of pressure measurements as well as accuracy of setting the tube distance from the wall was the same as in the series of measurements of velocity profiles in adjacent vicinity of the vessel wall.

EVALUATION OF MEASUREMENTS

From each measured m values of the angle of thread deviation β for given conditions in the mixed system, was calculated the average deviation angle as the arithmetic mean $\bar{\beta}$, and an estimate of the standard deviation σ_β of this quantity. As a rule the number of measurements m was 16, in some of the cases, values profoundly differing from the rest of values, were not considered. In Table I are, as an example, given values of these quantities calculated for measurements done with the propeller mixer of relative magnitude $d/D = 1/3$. Fig. 4 indicates schematically the flow direction in each measured point (angle $\bar{\beta}$) for given measured conditions at the speed of rotation 720 min^{-1}. Results obtained from measurements by the two-hole Pitot tube were evaluated in the following manner:

From measured values of the over-all and static pressures in a given point, characterized by the radial distance r from the wall, the dynamic pressure was calculated according to relation

$$p_{\mathrm{j,dyn}} = \Delta p_{\mathrm{j}} - \Delta p_{\mathrm{st}}\,, \quad [j = 1, 2] \tag{1}$$

and from this value was determined the absolute value of the local velocity vector by applying relation

$$|\mathbf{w}| = (2p_{\mathrm{j,dyn}}/\varrho K_{\mathrm{j}})^{1/2}\,, \quad [j = 1, 2]\,, \tag{2}$$

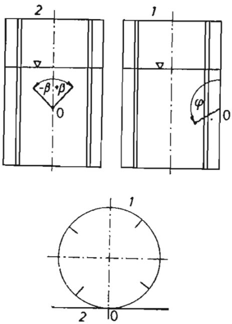

FIG. 3

Coordinate System at the Vessel Wall
1, *2* Corresponding sections.

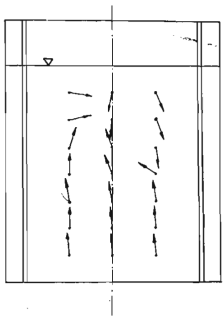

FIG. 4

Distribution of Angles $\bar{\beta}$ Along the Vessel Wall Between Two Neighbouring Baffles $d/D = 1/3$, $n = 720$ min^{-1}.

where value of the constant $K_j[j = 1, 2]$ was determined by calibration in the unit with a known velocity field[3] where it was found that $K_1 = 0{\cdot}855$, $K_2 = 0{\cdot}902$, and where values of these constants were found constant when the angle between the local velocity vector and the tube axis was not greater than 25°. Pressure data were considered for points on radial ray where was measured the over-all pressure $\Delta p_j[j = 1,2]$. Values of static pressure Δp_{st} were interpolated between the points of measurement and were extrapolated in the direction to the vessel wall where a static tube could not be placed. Absolute value of local velocity vector was related to the tangential tip velocity of the mixer by relation

$$W = |\mathbf{w}|/\pi dn\,. \tag{3}$$

The profiles of quantities W along the radial ray are, for positions *6*, *11*, and *16* given in Fig. 5. For points of measurements closest to the wall ($r = 0{\cdot}40$ mm) was considered a correction for shifting of this point in the direction from the wall[7] according to

$$\delta r = 0{\cdot}132 d_2 + 0{\cdot}081 d_1\,. \tag{4}$$

TABLE I

Values of Angle $\bar{\beta}$

Propeller mixer $s = d$, $d/D = 1/3$.

Position	$n = 500$ $\bar{\beta}$ deg	$[\text{min}^{-1}]$ σ_β deg	$n = 720$ $\bar{\beta}$ deg	$[\text{min}^{-1}]$ σ_β deg	$n = 840$ $\bar{\beta}$ deg	$[\text{min}^{-1}]$ σ_β deg
1	−3·7	2·4	−3·4	3·1	−2·2	2·6
2	−8·4	3·7	−8·4	3·7	−8·1	3·3
3	−6·9	3·1	−7·8	3·2	−6·6	2·9
4	−2·2	3·1	−3·4	2·6	−3·1	2·6
5	−8·1	3·7	−7·8	3·7	−8·4	3·2
6	−5·0	2·9	−4·7	3·1	−5·9	2·5
7	−5·0	3·4	−5·6	3·4	−7·2	3·2
8	−11·3	4·4	−11·9	4·1	−11·3	3·4
9	−11·9	7·1	−12·5	4·5	−10·0	3·7
10	−7·5	4·1	−6·3	3·3	−8·4	3·7
11	−18·4	8·1	−19·7	9·5	−19·1	8·8
12	−37·0	22·7	−32·5	18·2	−38·6	21·5
13	14·0	5·9	15·0	5·6	15·6	7·5
14	−10·9	10·7	−18·0	9·5	−26·6	21·2
15	−205·3	53·4	−201·3	52·9	−185·6	62·8
16	69·1	38·1	49·1	30·4	65·3	36·8
17	−193·1	52·5	−184·7	54·7	−169·7	51·9
18	−197·5	51·6	−203·4	53·9	−200·6	52·5
19	86·8	42·1	98·1	40·5	121·6	49·3
20	−179·7	48·2	−170·1	48·6	−174·7	46·5
21	−208·4	56·6	−208·1	55·9	−214·1	56·2

From the measured values of over-all pressures in individual taps of a three-hole oriented Pitot tube was calculated the absolute value and the angle characterizing the direction of local velocity vector by procedure described in one of previous studies[3] of this series. Absolute value of local velocity vector $|\mathbf{w}|$ was thus obtained for each position at the wall (see Fig. 1) and, beside the already known angle $\bar{\beta}$, also the value of angle φ (see Fig. 3). From these values was then determined the component w_x of the velocity vector in the plane tangential to the cylindrical area in a given point at the vessel wall by use of relation

$$w_x = |\mathbf{w}| \cos \varphi \,. \tag{5}$$

Table II gives, as an example, values of the mentioned quantities for a propeller type mixer of relative size $d/D = 1/3$.

On basis of the found flow directions in each point at the wall which did not practically depend either on the size, or the rotational speed of the mixer, was proposed the universal idealized flow pattern at the vessel wall between two neighbouring baffles in a form of a hypothetic mean tra-

TABLE II

Velocity Distribution Along the Vessel Wall Between Two Neighbouring Baffles
Propeller mixer $s = d$, $d/D = 1/3$.

Position	$n = 500$ [min^{-1}]			$n = 720$ [min^{-1}]			$n = 840$ [min^{-1}]		
	φ deg	$\|\mathbf{w}\|$ ms^{-1}	w_x ms^{-1}	φ deg	$\|\mathbf{w}\|$ ms^{-1}	w_x ms^{-1}	φ deg	$\|\mathbf{w}\|$ ms^{-1}	w_x ms^{-1}
1	1·5	0·31	0·31	2·5	0·53	0·53	2·5	0·64	0·64
2	8·5	0·21	0·20	7·0	0·55	0·54	7·5	0·60	0·59
3	7·0	0·39	0·38	6·5	0·61	0·60	6·5	0·63	0·62
4	2·0	0·36	0·36	6·5	0·57	0·56	6·0	0·61	0·60
5	4·0	0·33	0·33	8·0	0·65	0·64	7·0	0·65	0·64
6	6·5	0·29	0·28	7·5	0·40	0·39	7·0	0·49	0·48
7	6·5	0·29	0·28	6·5	0·55	0·54	6·5	0·58	0·57
8	8·0	0·19	0·19	7·0	0·40	0·39	4·0	0·58	0·58
9	7·0	0·31	0·30	8·0	0·33	0·32	4·5	0·46	0·46
10	5·0	0·26	0·26	3·5	0·44	0·44	8·0	0·48	0·47
11	6·5	0·14	0·14	8·0	0·33	0·32	7·5	0·37	0·36
12	69·0	0·23	0·08	72·5	0·23	0·05	56·0	0·48	0·16
13	5·0	0·20	0·20	8·0	0·24	0·23	10·0	0·31	0·37
14	10·0	0·11	0·11	6·0	0·16	0·16	8·0	0·21	0·21
15	165·0	0·06	0·06	156·0	0·14	0·10	155·0	0·21	0·17
16	25·0	0·14	0·11	25·0	0·17	0·14	25·0	0·20	0·18
17	15·0	0·08	0·07	12·5	0·08	0·07	12·0	0·08	0·07
18	167·5	0·12	0·10	167·5	0·09	0·07	167·5	0·13	0·11
19	45·5	0·13	0·07	37·0	0·12	0·08	45·0	0·13	0·08
20	167·5	0·04	0·03	167·5	0·01	0·01	170·0	0·05	0·05
21	167·5	0·06	0·04	167·5	0·01	0·01	167·5	0·06	0·06

TABLE III

Positions at the Wall Taken into Account for Calculation of Angle γ_k

Angle number γ_k	Number of position at the wall
k	l
1	1, 2, 3, 4, 5, 6, 7, 8, 9
2	10, 11, 13, 14,
3	16, 19
4	15, 18, 20, 21

jectory of the liquid particle. Direction of the particle path was found as a mean value of the direction of the local velocity vector (angles β) in different points at the vessel wall. The determined angles γ_k for particular parts of the wall are obvious from Fig. 6. These angles were determined by relation

$$\gamma_k = (1/N) \sum_{l=1}^{N} \beta_l \,, \quad [k = 1, 2, 3, 4] \,. \tag{6}$$

Position numbers l (see Fig. 1) which were taken into consideration for calculation of the corresponding angle value γ_k are indicated in Table III.

RESULTS AND DISCUSSION

VELOCITY FIELD AT THE VESSEL WALL

The properties of radial profiles of absolute value of local velocity vector at the vessel wall (see Fig. 5) are similar to that of velocity profile of the liquid flow at the surface of a solid body. In adjacent vicinity of the wall is apparent the velocity gradient and at a certain distance from the wall, the flow velocity can be considered constant, *i.e.* the flow is no more affected by vicinity of the solid body. For the six-bladed paddle mixer it was found that in all measured cases (positions at the wall) the velocity could be considered constant at radial distance from 3 to 6 mm from the wall. Analogous conclusions obtained Askew and Beckmann[1] for the system dimensionally close to the system used in this work, only the mentioned authors used the propeller mixer. Therefore the already mentioned distance 4 mm from the wall of the three-hole oriented tube was chosen. However this velocity varies from one position to another especially in the upper half of the vessel. Its value toward the liquid surface decreases which also is confirmed by results obtained in papers[10,11] on measurements of velocity field in a mixed charge. In the lower half of the wall this velocity as well as its direction does not change their values which is in agreement with the results of the cited paper[1].

The shape of velocity profile at the vessel wall varies with the point of measurement: In the position with a higher velocity is the velocity profile steeper because the tube distance is in all measured points the same. In agreement with the known fact *e.g.*[1,3,10,11] the liquid velocity in adjacent vicinity of the vessel wall is also directly proportional to the rotational speed of the mixer and the measurements made at different rotational speeds can be thus intercompared. It follows from our measurements (see also Fig. 5) that the range of values of dimensionless velocities obtained at speeds 450 and 600 min^{-1} is not greater than 10%.

The velocity field at the wall of the mixed vessel with radial baffles at the use of an radial mixer, can be practically considered flat, because the value of angle φ determined experimentally in individual points did not exceed the value of 25° and in the lower half of the wall even the value of 10°. Only in three points (positions *12*, *14*, and *19*) is value of the angle systematically higher. In the region of positions *12*

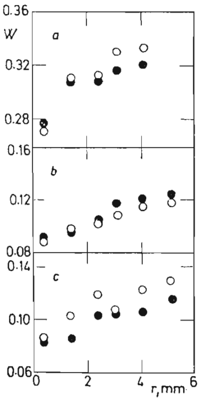

FIG. 5

Radial Profile of Dimensionless Absolute Value of Local Velocity Vector at the Vessel Wall (six bladed propeller mixer with inclined blades)

a Position *6*, *b* position *11*, *c* position *16* ○ $n = 450\ min^{-1}$, ● $600\ min^{-1}$.

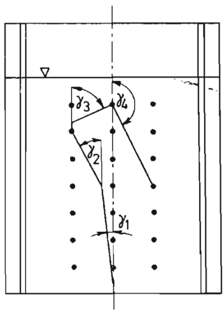

FIG. 6

Flow Pattern at the Vessel Wall Between Two Neighbouring Baffles. Propeller Mixer $s = d$

$\gamma_1 = -6°$, $\gamma_2 = -22°$, $\gamma_3 = 75°$, $\gamma_4 = -217°$.

and *14* a contact of two streams takes place (upward and downward), in the region of position *19* there shows at the chosen rotation direction a vortex effect on the surface behind the baffle. The proposed flow pattern at the wall between two baffles (Fig. 6) expresses the mean path of the liquid particle entering the region of the wall at the vessel bottom. The flow at the wall can be, according to this pattern, divided into two parts. In the lover half, the flow is practically vertical, upwards with a small effect of mixer rotation (it is mainly eliminated by baffles). In the upper half of the wall the flow pattern is more complicated – because there exists the effect of practically still liquid surface and also the effect of negative pressure behind the baffle. In mentioned two parts of the flow-pattern the absolute values of flow velocities significantly differ. In the upper half of the wall the absolute value of velocity vector is substantially lower than in the lower half because the charge is already by the sucction effect of the mixer reversed *i.e.* downward and simultaneously to the system axis. This explains among others, the increase of flow velocity between the measured positions *21*, *18* and *15*, *i.e.* opposite to the flow direction in the lower part of the vessel.

The cited authors[1] measured the velocity field in vicinity of the wall in the lower half of the vessel. From comparison of their results with the results of this work it follows that at the same tangential tip velocity of the mixer, the absolute values of velocity vectors were three-times higher than velocities experimentally determined in corresponding points of the system by the procedure presented in this work. When we compare these velocities with the mean liquid velocity flowing through the rotating mixer as defined by relation

$$\bar{w}_{\mathrm{M}} = 4\ \mathrm{K}_{\mathrm{p}} n d / \pi \tag{7}$$

then for the value of the flow criterion[8]

$$\mathrm{K}_{\mathrm{p}} \equiv \dot{V}/nd^3 \approx 0{\cdot}50\,, \quad [\text{propeller mixer } s = d,\ d/D = 1/3] \tag{8}$$

the velocity at the wall, determined by the mentioned authors, is more than two-times larger the velocity w_{M}, while the velocity at the wall in its lower half, determined in this work, equals to approx. 70% of the value of velocity w_{M}. Maximum velocity in the rotating propeller mixer under these conditions[12] is more than twice as large as the mean velocity w_{M}, however the ray streaming from the mixer widens and its mean and maximum velocities decrease. At the vessel wall is then the cross-sectional area of the upward flow practically same as the cross-sectional area of the downward flow at the bottom and consequently it cannot reach the same velocity values as at the exit from the mixer. This fact was experimentally confirmed several times[3,11–15]. Objections to the results published by the cited authors[1] are further supported by the fact that they do not describe the method of measurements of static pressure at the wall, though this quantity as we have found forms up to 30% of the value of the over-all pressure determined by the Pitot tube.

Effect of Velocity Field at the Wall on Heat Transfer from Wall into Charge

Informations obtained from measuring the velocity field in vicinity of the wall can be used for considerations concerning heat transfer between the vessel wall and the charge. In this process the intensity of transfer is significantly affected by the flow in adjacent vicinity of the wall. At the assumption that in the mixed charge the temperature is balanced by convective flow very quickly, then for the heat transfer rate from the wall into the charge (or *vice versa*) is decisive the heat transfer between the wall and the liquid in adjacent vicinity of the wall. On basis of knowledge of the velocity field it is possible in this region to estimate values of the heat transfer coefficient. Askew and Beckmann[1] considered the vessel wall to be a plane and on basis of analogy between the heat and momentum transfer in the boundary layer along a flat plate calculated the value of local heat transfer coefficient from the wall into the charge in a distance from the bottom corresponding to the vertical distance of the mixer from the bottom. In this way calculated value of the heat transfer coefficient was in agreement with the value directly measured in the identical point of the considered system which was taken as a sufficient proof of the proposed theoretical model. The proposed model was based on the idea of the boundary layer which starts forming in the corner between the vessel wall and the bottom. We tried earlier[16] to extend this idea on the whole wall, *i.e.* for calculation of the mean heat transfer coefficient along the whole wall (or in its significant parts) from the knowledge of velocity field at the wall. This experiment was, however, not successful because the idea of beginning of the boundary layer in the corner between the vessel wall and the bottom does not correspond to conditions in the mixed system. The same applies for the velocity values determined in vicinity of the wall which significantly differ from the values given in the cited paper[1] (see above) and which prevents even reproduction of the results claimed by the cited authors. Mechanism of heat transfer in the lower part of the vessel wall is obviously more complex than corresponds to the turbulent boundary layer at the flat plate and thus it is necessary to consider a fully developed turbulent field along the whole wall as well as at the vessel bottom[14]. From knowledge of the velocity field at the wall assumptions can be made on the heat transfer intensity from the wall into the charge in different parts of the wall. Since on the basis of analogy between the heat and momentum transfer from the tube wall into the flowing liquid is valid the proportionality

$$\tau/\varrho = \text{const.}\,(\dot{q}/c_p\varrho)\,, \tag{9}$$

than by multiplying both sides of equation (9) by area A on which simultaneously acts the shear stress τ and over which the heat of density $\dot{q}$ is transferred, we obtain after arrangement

$$\tau A = \text{const.}\ (\dot{Q}/c_p)\,. \tag{10}$$

When we express the thermal flow $\dot{Q}$ by the Newton law for cooling, we can write

$$F = A\tau = \text{const.}\ (\alpha A\ \Delta t/c_p)\,. \tag{11}$$

Expression on the left side of Eq. (*11*) represents the total shear force on the area A and expression on the right side is directly proportional to the over-all heat transferred per unit of time through the same area. If the lower half of the vessel has index I and the upper half index II, than by comparison of both sides of Eq. (*11*) for the mentioned areas we get

$$F_{II}/F_I = \tau_{II}A_{II}/\tau_I A_I = \alpha_{II}A_{II}\ \Delta t_{II}/\alpha_I A_I\ \Delta t_I\,. \tag{12}$$

When we also take into account the assumption (mentioned above) that temperature in the charge is quickly balanced by convective flow, and if we consider the temperature of the whole wall to be constant, then also the temperature difference Δt between the wall and the charge along the whole wall is constant. Intensity of heat transfer from each part of the wall into the charge dependes only on the total shear force along the considered part and we can thus compare how the given part of the vessel wall contributes to heat transfer with the charge.

As it is obvious from results of measurements of directions of velocity vectors along the wall (see Fig. 6), that flow in the lower half of the vessel can be considered as a flow through a circular pipe as the velocity vector has the direction practically identical with the axis of system. In the upper part is the character of flow substantially different as the liquid flows roughly along the wall. For calculation of the total shear force was thus in this region considered the flow along the smooth flat plate with the origin of the boundary layer at the corner of the vessel bottom. Table IV gives ratios of total tangential forces in the upper and lower halves of the vessel wall for experiments made in this work, *i.e.* calculated from the known distribution of the velocity component w_x (see Eq. (*5*)) along the wall. For the lower part of the wall the total shear force F_I was calculated for the case of turbulent flow in the pipe by use of relation

$$F_I \equiv \sum_{k=1}^{N_I} \tau_k A_k\,, \tag{13}$$

where

$$\tau_k = (f_k/2)\, w_{xk}^2 \varrho\,. \tag{14}$$

The shear stress τ_k corresponding to area A_k was calculated from the mean value of velocity w_{xk} over the area A_k. In area A_1 were situated points *1–3*, in area A_2 points *4–6*, in area A_3 points *7–9*, and in area A_4 points *10–12* (see Fig. 1). Friction factor f_k was calculated for the flow through a smooth circular pipe by Blasius relation

$$f_k = 0{\cdot}079(w_{xk}D/\nu) - 0{\cdot}25\,. \tag{15}$$

For the upper part of the wall the total tangential force F_{II} was calculated for turbulent boundary layer along the smooth flat plate by use of relation

$$F_{II} = L_{II}\int_{l_{II1}}^{l_{II2}} \tau_{II}(w_{xII}, l_{II})\,dl_{II}\,, \tag{16}$$

where

$$L_{II} \equiv A_{II}/(l_{II2} - l_{II1}) \tag{17}$$

and where

$$\tau(w_{xII}, l_{II}) = [f(w_{xII}, l_{II})/2]\,w_{xII}^2(l_{II})\,\varrho\,. \tag{18}$$

The friction factor was calculated for turbulent flow along the smooth flat plate[4] by Blasius relation

$$f(w_{xII}, l_{II}) = 0{\cdot}0292\,(w_{xII}(l_{II})\,l_{II}/\nu) - 0{\cdot}20\,. \tag{19}$$

The integration path l_{II} was given by the proposed particle trajectory in the given part of the wall (see Fig. 6), when the origin of the boundary layer ($l_{II} = 0$) was situated at the corner between the bottom and the wall, *i.e.* at the origin of the idealized trajectory at the wall.

From Table IV it follows that the total tangential force in the upper half of the wall equals 22–27% of value of the total shear force in the lower half. This result leads to conclusion that in the lower part of the wall the intensity of heat transfer is 4–5 times higher than in the upper half. The upper half thus contributes only very little to the heat transfer between the charge and the wall. This fact results also from me-

TABLE IV

Ratios of Total Shear Forces at the Wall in the Upper and Lower Half of Vessel

d/D (–)	n min^{-1}	F_{II}/F_I %	d/D (–)	n min^{-1}	F_{II}/F_I %
1/5	1 300	24·5	1/4	1 750	23·0
1/5	1 700	25·4	1/3	500	27·0
1/5	2 100	21·9	1/3	720	22·0
1/4	1 050	21·8	1/3	840	22·5
1/4	1 400	22·4			

asurements of heat transfer from the wall into the charge. For the system geometrically similar to the system used in this work, at turbulent conditions in the mixed system, Strek and coworkers[13] give the relation

$$\mathrm{Nu_M} = 0{\cdot}342\ \mathrm{Re_M}^{2/3}\ \mathrm{Pr}^{1/3}\,, \quad [\text{propeller mixer } s = d,\ d/D = h/D = 1/3] \tag{20}$$

and for the system in which $H = 1{\cdot}5D$ Kupčík[14] gives a correlation equation

$$\mathrm{Nu_M} = 0{\cdot}215\ \mathrm{Re_M}^{2/3}\ \mathrm{Pr}^{1/3}\,, \quad [\text{propeller mixer } s = d,\ d/D = h/D = 1/3] \tag{21}$$

obtained also from experimental studies of heat transfer. By comparing Eq. (*20*) and (*21*) we can find that at identical values of $\mathrm{Re_M}$ and Pr, and at identical values of vessel diameter and thermal liquid conductivity λ, the heat transfer coefficient will increase for approx. 50% if the height of liquid in the vessel decreases from the value $H = 1{\cdot}5D$ to $H = D$. Amount of heat transferred is for both cases practically constant as for $H = 1{\cdot}5D$ is the heat transfer area by 50% larger than for $H = D$. Decreasing of the vessel diameter need not improve conditions of heat transfer into the charge as the effect of velocity field at the wall can practically eliminate such change. This effect can then be studied not only by direct study of heat transfer but also by study of the velocity field in the considered region and of its changes at different geometrical conditions of the system.

LIST OF SYMBOLS

A area of vessel wall [m^2]
c_p specific heat [kcal kg^{-1} deg^{-1}]
D vessel diameter [m]
d mixer diameter [m]
d_1 inside diameter of the Pitot tube [m]
d_2 outside diameter of the Pitot tube [m]
F total shear force [N]
H height of liquid above the vessel bottom when at rest [m]
h height of mixer center above the vessel bottom [m]
i summation index
j summation index
K_j resistance coefficient of j-th Pitot tube
k summation index
L equivalent wall width [m]
l length of particle trajectory at the wall [m]
m number of measurements of angle β in the given point
N number of points at the wall for determination of γ_k
N_I number of areas in the lower part of wall for determination of force F_I
n rotational speed of mixer [s^{-1}]
Δp_{st} static pressure [Nm^{-2}]

$p_{j,dyn}$ dynamic pressure in j-th tube tap [Nm^{-2}]
Δp_j difference of over-all pressure in j-th tube tap [$N m^{-2}$]
$\dot{Q}$ amount of heat transferred per unit of time from the vessel wall into the charge [$kcal h^{-1}$]
$\dot{q}$ density of heat flow from vessel into charge [$kcal m^{-2} h^{-1}$]
r radial distance from vessel wall [m]
s pitch of mixer blades [m]
t temperature [deg]
$\dot{V}$ volumetric mixer capacity [$m^3 s^{-1}$]
$|\mathbf{w}|$ absolute value of local velocity vector [$m s^{-1}$]
w_x component of velocity vector $\mathbf{w}$ in plane tangential to cylinder surface [$m s^{-1}$]
$\bar{w}_M$ mean velocity through rotating mixer [$m s^{-1}$]
α mean heat transfer coefficient over area of the vessel wall [$kcal^{-2} h^{-1} deg^{-1}$]
β flow angle in tangential plane in given point [deg]
γ angle of particle trajectory at vessel wall [deg]
λ thermal conductivity of charge [$kcal m^{-1} h^{-1} deg^{-1}$]
ϱ density of charge [$kg m^{-3}$]
η dynamic viscosity of charge [$kg m^{-1} s^{-1}$]
ν kinematic viscosity of charge [$m^2 s^{-1}$]
δ_a standard deviation of quantity a
φ angle between the local velocity vector and tangential plane [deg]
τ shear stress at the wall [$N m^{-2}$]
f friction factor
$K_p \equiv \dot{V}/nd^3$ flow rate number
$Nu_M \equiv \alpha D/\lambda$ Nusselt number
$Pr \equiv \nu/\lambda$ Prandtl number
$Re_M \equiv nd\varrho/\eta$ Reynolds number
$W = |\mathbf{w}|/\pi dn$ absolute, dimensionless value of local velocity vector

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